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String Breaking in Two-Dimensional QCD*

K. Hornbostel

Southern Methodist University, Dallas, TX 75275

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Abstract

I present results of a numerical calculation of the effects of light quark-antiquark pairs on the linear heavy-quark potential in light-cone quantized two-dimensional QCD. I extract the potential from the $Q\bar{Q}$ component of the ground-state wavefunction, and observe string breaking at the heavy-light meson pair threshold. I briefly comment on the states responsible for the breaking.

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1 Introduction

Pure Quantum Chromodynamics exhibits linear confinement between charges at large distances, binding them with a gluonic string, as observed some time ago in lattice simulations of the potential between static quarks [1]. Stretching the string sufficiently in the presence of light quarks should produce quark-antiquark pairs, breaking the string and screening the charges; attempts to clearly demonstrate this effect in lattice QCD are ongoing [2]. The purpose for this study is to determine whether string breaking can be observed in a relatively simple model, two-dimensional QCD [3], using a very different numerical scheme: solving for eigenstates of the light-cone (LC) Hamiltonian for the system confined to a box [4].

Early interest in LC quantization focused on high-energy scattering, due in part to the boost invariance of LC wavefunctions and their direct connection to parton distribution functions [5]. However, the suggestion of a simplified vacuum and successes with two-dimensional models generated some recent interest in using this scheme to perform nonperturbative calculations of static properties in QCD [6]. In addition to their connection to parton distributions, LC wavefunctions have a well-defined heavy-quark limit [7] and a simple connection to quantum mechanical wavefunctions, which I will make use of in extracting the heavy-quark potential below.

In this formalism, one initializes fields and their commutators at equal LC time, $x^+ \equiv x^0 + x^3 = 0$. The Hamiltonian $P^- \equiv P^0 - P^3$ evolves states in x^+ , while the LC momentum $P^+ \equiv P^0 + P^3$ is kinematic and conserved in interactions. Diagonalizing P^- in the space of states defined by acting with creation operators on the vacuum produces the masses and wavefunctions for hadrons and multiple-hadron states. All hadronic matrix elements may be expressed as integrals over these wavefunctions.

In order to accomplish this numerically, I confine the system to a box of length L in x^- and impose antiperiodic boundary conditions on the fields. As a result, each quark carries k^+ in odd half integer units $2\pi/L$. Because all k^+ are positive and must sum to the total P^+ , the discrete momentum $K \equiv (L/2\pi)P^+$ determines both the resolution of the wavefunctions and also controls the number of quanta possible in a state. $K \rightarrow \infty$ defines the continuum limit.

The program which simulates this system allows for any number of colors and flavors. Once given conserved quantities such as baryon number, total momentum, and individual flavor numbers, it generates all states consistent with these, though it allows for additional restrictions. It computes the Hamiltonian in this basis and diagonalizes it, generating the complete set of masses and wavefunctions. That it produces all states is both an advantage and curse, as the number of states grows roughly exponentially with K . See J. Hiller's contribution to these proceedings [8] and references therein for a discussion of an intelligent way to focus on the lowest lying states.

While fully relativistic, the use of a Hamiltonian, a Fock space description of states, and wavefunctions makes it similar in form to nonrelativistic many-particle quantum mechanics. A final similarity is the ability to work with a

simple vacuum. Roughly, because all particles carry positive k^+ , it is not possible for their momentum to sum to the zero P^+ of the vacuum; therefore, the vacuum has no particles. The true situation is more involved, and whether this argument holds depends on what is being computed. Extracting vacuum properties in particular can be subtle [9]. But when applicable it is an enormous simplification.

One of the advantages of LC quantization absent in the older infinite momentum frame formulation is that it selects a quantization surface rather than a specific reference frame. As a result, it has a well-defined nonrelativistic limit for heavy quarks, in which LC wavefunctions go smoothly into the usual wavefunctions of nonrelativistic quantum mechanics. Because I wish to extract the potential felt by two nearly static quarks, I include only one heavy flavor. Heavy in two-dimensional QCD means that m_Q is large compared to the dimensionful coupling g ; in four dimensions, the comparison would be to Λ_{QCD} .

To see the Schrödinger equation these satisfy, consider the projection of the LC equation

$$M^2 |\phi\rangle = P^+ P^- |\phi\rangle \quad (1)$$

onto the $|Q\bar{Q}\rangle$ subspace. States in this subspace have the form

$$\int_0^1 dx \phi_{Q\bar{Q}}(x) b^{c\dagger}(xP^+) d_c^\dagger((1-x)P^+) |0\rangle \quad (2)$$

where x gives the fraction of P^+ carried by the quark with probability $|\phi_{Q\bar{Q}}(x)|^2$, and the color index c contracts to form a singlet. While the complete wavefunction contains states with extra $Q\bar{Q}$ pairs, these are suppressed by factors of $1/m_Q$. The result is 't Hooft's equation [3]:

$$M^2 \phi_{Q\bar{Q}}(x) = m_Q^2 \left[\frac{1}{x} + \frac{1}{(1-x)} \right] \phi_{Q\bar{Q}}(x) - \frac{g_N^2}{\pi} \int_0^1 dy \frac{\phi_{Q\bar{Q}}(y) - \phi_{Q\bar{Q}}(x)}{(y-x)^2}. \quad (3)$$

Here $g_N^2 \equiv g^2(N^2 - 1)/2N$, and the principal value prescription defines the integral. Though restricted to the two quark sector, this equation is still fully relativistic; in fact, it describes mesons to leading order in large N .

In the large- m_Q limit, the momentum fraction carried by the quark,

$$x \equiv \frac{k^+}{P^+} = \frac{E(k) + k}{E(P) + P} \approx \frac{1}{2} + \frac{q}{2m_Q} \quad (4)$$

to lowest order in the relative momentum q , and the LC wavefunction $\phi_{Q\bar{Q}}(x)$ becomes sharply peaked around $1/2$. Introducing a nonrelativistic momentum-space wavefunction

$$\psi(q) \equiv \phi_{Q\bar{Q}}(1/2 + q/2m_Q) \quad (5)$$

focuses on deviations from this peak. Defining the nonrelativistic energy $E \equiv M - 2m_Q$ and reduced mass $\mu \equiv m_Q/2$, extending q from $\pm m_Q$ to infinity,

and Fourier transforming $\psi(q)$ to the relative position r turns Eq. (3) into the Schrödinger equation [10, 11, 4]

$$\left[-\frac{1}{2\mu} \partial_r^2 + \frac{1}{2} g_N^2 |r| \right] \psi(r) = E \psi(r) . \quad (6)$$

The relation

$$\int_{-\infty}^{\infty} \frac{dq}{q^2} [e^{-iqr} - 1] = -\pi |r| . \quad (7)$$

converted the integral over q into a linear potential in position with string tension $g_N^2/2$. The solutions are Airy functions; see Ref. [10] for details.

The computer code produces masses and LC momentum-space wavefunctions on whose Fourier transforms Eq. (6) holds in the large- m_Q limit. Therefore, computing $\psi(r)$ and $\partial_r^2 \psi(r)$ by transforming $\psi(q)$ and $q^2 \psi(q)$ respectively and defining [12]

$$V_{Q\overline{Q}}(r) \equiv \frac{2\mu E \psi(r) + \partial_r^2 \psi(r)}{\mu g_N^2 \psi(r)} \quad (8)$$

should reproduce a linear potential in that limit. In particular, the degree to which $V_{Q\overline{Q}}$ conforms to $|r|$ indicates how accurately a nonrelativistic Schrödinger equation with a static potential describes this meson. Figures 1 and 2 display potentials extracted in this manner for two values of g_N/m_Q . These reconstruct $g_N^2|r|/2$ fairly reliably at the relatively modest momentum $K = 15$, although the effects of discretization appear in the turnover of $V_{Q\overline{Q}}$ near its edges.

2 String Breaking

In one spatial dimension, there are no transverse directions in which the electric field may spread. As a result, the Coulomb potential is linear; in $A^+ = 0$ gauge, it is all that remains of the gauge field. Because the string is present in the classical Hamiltonian, all the work on the quantum level goes into breaking it, making it a convenient laboratory to study this effect. To do so, I introduce an additional light-quark flavor and study the effect the presence of extra light-quark pairs has on the potential defined in Eq. (8). More specifically, because the program includes all states consistent with quantum numbers specified, I include three flavors: two different heavy quarks with identical mass and one light quark, but I fix their respective flavor numbers to 1, -1 and 0. This ensures that every state contains at least one heavy $Q\overline{Q}$ pair.

Figures 1 and 2 display preliminary results for the potential in the presence of light quarks at couplings of $g_N/m_Q = .164$ and $.247$ and a light quark mass of $m_q = .001/m_Q$. The total discrete momentum $K = 15$ for these, leading to 15 points in the $Q\overline{Q}$ wavefunction. I used an invariant mass cutoff to exclude irrelevant extra heavy-quark pairs, and also restricted by hand the state space to a single additional $q\overline{q}$ pair in order to keep the number of states manageable; typically, about 1300. Testing this truncation by allowing additional pairs at lower K had little effect, suggesting that the production of a single light-quark

pair was predominantly responsible for breaking the string. The data displayed came from a run of roughly half an hour on a DEC alpha 3000/700 workstation. Along with the ground states from which these potentials were extracted, the program produced the entire set of roughly 1300 masses and wavefunctions for this system. The LC Hamiltonian has a relatively simple structure, with the coupling and masses appearing as overall factors before the free and interacting terms. Almost all of the computational effort goes into producing a complete orthonormal set of states and evaluating the Hamiltonian matrix. Therefore, by storing separately the free and interaction matrices, reproducing the spectrum for different couplings and masses is essentially free.

Both the linear potential for the pure $Q\bar{Q}$ system and its breaking in the presence of light quarks are clearly evident. The horizontal line in each plot indicates the additional cost in energy to produce a pair of heavy-light mesons, showing that the string breaks about where expected. To obtain these, I computed the mass of the lightest $Q\bar{q}$ mesons with the same parameters but half the value of K . This approximates that carried by each heavy-light meson produced in the $Q\bar{Q}q\bar{q}$ system.

The ground state includes in its wavefunction not only the $|Q\bar{Q}\rangle$ states from which $V_{Q\bar{Q}}$ is extracted, but also higher-Fock states $|Q\bar{Q}q\bar{q}\rangle$ with additional light quark pairs. By examining their magnitude, it is possible to gain some insight into which states are most responsible for breaking the string. For the $g_N/m_Q = .247$ case, states in which each light quark forms a singlet with a heavy quark and the LC momentum is carried almost entirely and equally by the heavy quarks are predominant. This is in accord with the tendency for Fock-state constituents to move with equal velocity [13]. These account for approximately .5% of the total probability. Figure 3 shows the difference between the position-space wavefunctions for $g_N/m_Q = .247$, before and after the inclusion of light quarks. These exhibit the spread away from $r = 0$ which results from charge screening by the light quarks.

In these figures, I computed $V_{Q\bar{Q}}$ from the ground state wavefunctions. Note, however, that Eq. (8) applies equally to excited states. $V_{Q\bar{Q}}$ extracted from these is consistent but significantly degraded.

There are some tradeoffs evident in these preliminary results. The larger the coupling, the more dramatic the breaking and longer the extent of the wavefunction. But the larger coupling leads to a less relativistic $Q\bar{Q}$ system, and a wider wavefunction in momentum, making it more susceptible to finite box size errors. It also increases the importance of additional $q\bar{q}$ pairs thus far neglected.

3 Future Work

In addition to simply increasing K , there are a number of ways of improving the accuracy of this study. Fitting the wavefunctions to physically motivated continuous functions would ameliorate some of the discretization and finite-size errors. The range of $V_{Q\bar{Q}}$, which is limited by this method to the width of the

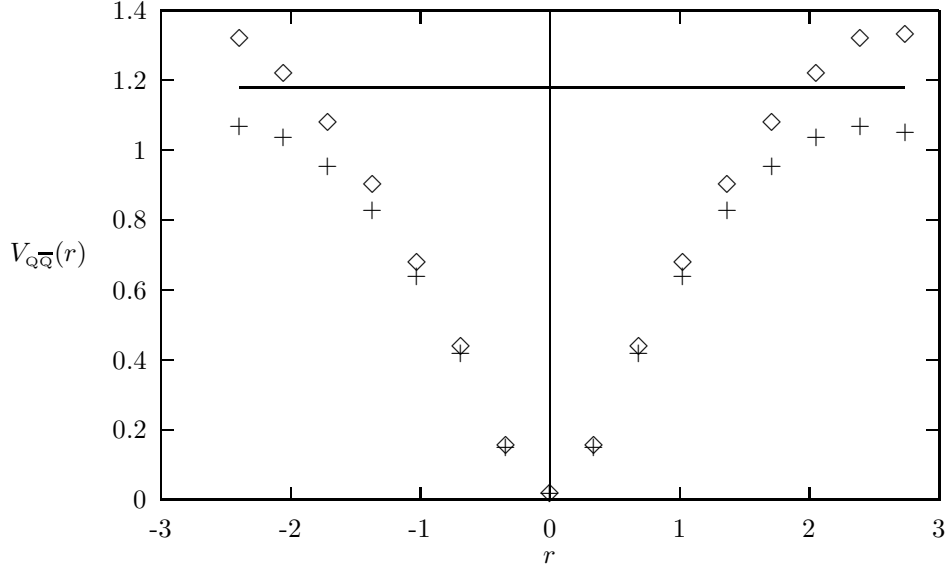


Figure 1: The potential defined in Eq. (8) at $g_N/m_Q = .164$ for two heavy quarks (\diamond), and after the inclusion of one light flavor with $m_q/m_Q = .001$ (+). The discrete momentum $K = 15$ for both cases. The horizontal line gives the additional energy $2M_{Qq} - 2m_Q$ required to produce two heavy-light mesons, each computed at $K = 8$. The square root of the string tension $g_N^2/2$ fixes the units for $V_{Q\bar{Q}}$ and r .

wavefunction, could also be extended with the use of static sources, though this would mean abandoning the momentum conservation currently built into the code. Finally, an intelligent method for importance sampling of states, and perhaps the use of effective interactions to replace states excluded based on energy, would certainly improve the efficiency of this program.

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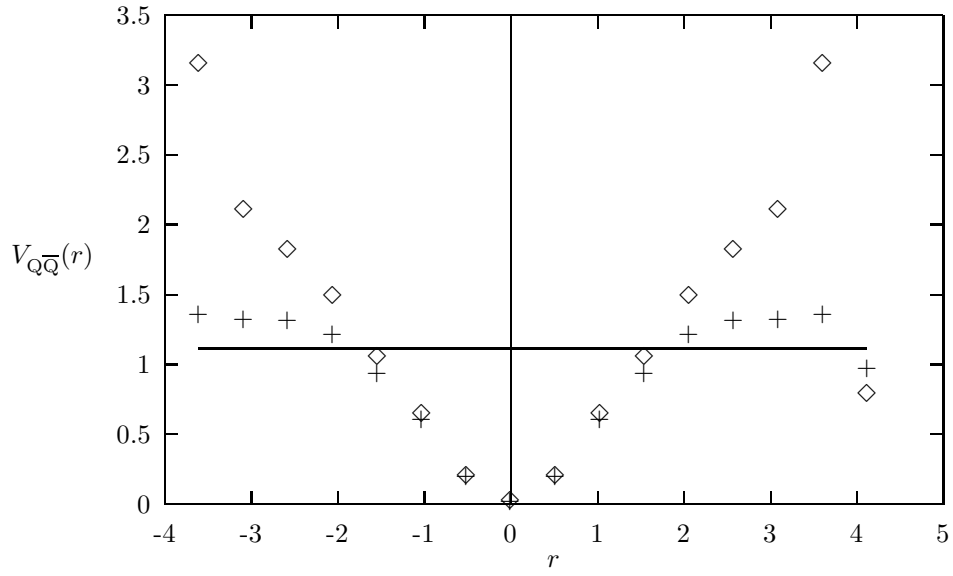


Figure 2: The potential defined in Eq. (8) at $g_N/m_Q = .247$ for two heavy quarks (\diamond), and after the inclusion of one light flavor with $m_q/m_Q = .001$ ($+$). The discrete momentum $K = 15$ for both cases. The horizontal line gives the additional energy $2M_{Qq} - 2m_Q$ required to produce two heavy-light mesons, each computed at $K = 8$. The square root of the string tension $g_N^2/2$ fixes the units for $V_{Q\bar{Q}}$ and r .

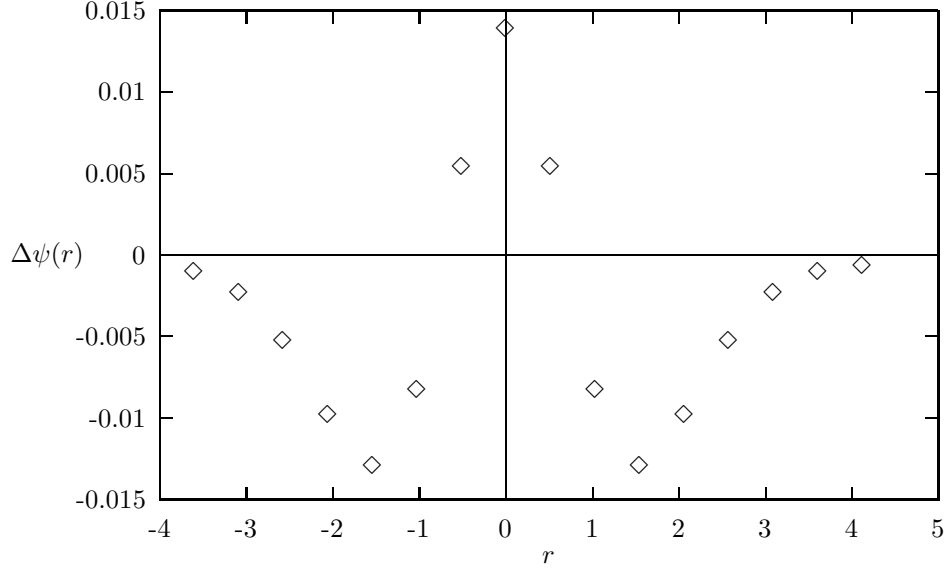


Figure 3: The difference between the position-space $Q\bar{Q}$ wavefunctions $\psi(r)$ computed at $g_N/m_Q = .247$ for two heavy quarks and after the inclusion of one light flavor with $m_q/m_Q = .001$. The square root of the string tension $g_N^2/2$ fixes the units for r .

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